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ABSTRACT

Examining how students reconstruct stories they've heard can give insights into why students often have difficulty understanding and retaining mathematics. Behavioral psychologists refer to the phenomenon of piecing together a series of events as "chaining." This paper argues that the cognitive capacity to reconstruct a whole contextual structure from a few entities within it belongs to even the slowest learners, and that this capacity to retain contextually related information can be activated to facilitate learning and retention of mathematics. The document presents an example of converting a fraction into a decimal in which traditional algorithms are learned through a contextual approach, leading to gains in efficiency and meaning in the acquisition of the skill. Teachers are asked to consistently present an internal contextual view of mathematics to learners. (MDH)

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Teaching Mathematics Through Context:

Unleashing the Power of the Contextual Learner

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TEACHING MATHEMATICS THROUGH CONTEXT:
UNLEASHING THE POWER OF THE CONTEXTUAL LEARNER

Learners in thousands of classrooms throughout our nation have difficulty understanding and retaining mathematics. But do they experience difficulty understanding and retaining stories? If they do not, then it might be useful to enquire concerning the basis for this phenomenon and its implications for mathematics education (Barrett, 1989).

Because teachers have so strongly encouraged students to memorize in order to retain mathematics, we might begin our inquiry by asking whether memorization is the means whereby even the slowest learners retain stories. This is clearly not the case since we know that children tell stories "in their own words," which are not the same as those they heard when they were told them originally. What, then, is this potent psychological functioning in all of us which makes retention of a story inevitable?

If you seriously engage yourself in an effort to recall a story you heard in the past, you will find that, in effect, your mind is "piecing it together" even as you tell it. Behavioral psychologists refer to this phenomenon as "chaining." You are able to do this as a consequence of the cause and effect relationships your mind perceived, as you heard it the first few times. You noted how a build up of circumstances would cause an

event which, in turn, would become part of a larger set of circumstances, which again would lead to another event and thus the story unfolded. Was not your effort to recall a story one of reconstruction rather than memorization?

What do the words "prince" and "slippers" trigger in your mind? These entities are experienced by your psyche as woven together in a contextual structure entitled "Cinderella." With the exception of a very small number of brain damaged individuals, every human mind is capable of reconstructing a whole story from a few entities within some context. This is what permits us to say we know the story and, more importantly, to know that we know it.

We all have had the experience of seeing a face and noting its familiarity, but never-the-less asking, "Who is that?" What psychological functioning occurs in order that we might reconstruct the person's identity? It is when that face triggers a few associated entities such as a certain high school, a certain incident, a certain other person. These entities trigger recollection of much additional, contextually related information. This contextual type of knowing permits its owners to declare with confidence and with the "certification" of internal authority, "I see!" "I understand!" "I know!" "This makes sense!"

By the age of three to four years, virtually all children have taught themselves a "foreign language." The average six year old already understands between 8,000 and 14,000 words (Carey, 1977). The first language that babies encounter is foreign in the sense that they did not know it at birth. Consider the fact that the meanings of many words they comprehend were neither looked up in a dictionary nor explained by Mom. Besides the meanings of words, they acquired meanings of phrases and sentences. Do you remember, as a teen-ager (or even quite recently), deciphering the meaning of a word or phrase based on how it was used in some conversation? Was this not an example of contextual learning? That children function in this manner to master a language at such an early age is as much true for slow learners as it is for quick ones. They function the same way since both use context as a means for acquiring language and retaining stories.

Whenever any human mind perceives, for the first time, a link between two entities, however irrelevant or insignificant each had become, this discovery is sufficiently significant to immediately attract the total preoccupation of the mind (however briefly) from all other conscious activities. For example, both the barking of the dog and the arrival of the mailman may have become insignificant to a child prior to the discovery of the dynamic between them, whereby the barking informs him/her that the mailman

has arrived. A link of any kind between two or more entities is of intrinsic and irresistible interest to all human beings (Bruner, 1963). Piaget's (1970) notion of equilibration as a search for a balance between a child's cognitive structure and his/her perceptions may be another appropriate frame of reference through which we might view this phenomenon. It is impossible for anyone to function in human society without this capacity.

It is clear from the above examples that the cognitive capacity to reconstruct a whole contextual structure from a few entities within it (a type of psychological functioning which is highly esteemed in academic society) belongs to the slowest learners you ever saw. This is one of our endowments. It belongs to us by virtue of merely being human. It ensures the retention of much contextually related information found in everyone's experiences.

Consequently, the reality is (Barrett, 1990),

> that every child was a competent contextual learner before going to school. The proof of this statement is that every child, with extremely few brain damaged exceptions, taught himself/herself a language at an early age and retained stories after hearing them a few times.

> It is well known that the predominant cognitive functioning necessary for such accomplishments is the construction of relationships within some context; otherwise called contextual learning. (p. 1)

If slow learners have the capacity to retain contextually related information, can it be activated to facilitate the learning and retention of mathematics? Is the view of context in a story also present, for example, in the teaching of long division or transforming a fraction into an equivalent decimal? Let us look at the traditional algorithm for transforming the fraction 5/8 to an equivalent decimal. See the example below.

$$8\overline{)5}$$

The teacher says, "Eight can't go into 5 once, so we must place a decimal point next to 5 and immediately place another decimal point above it." Why? "Just do it!" The teacher continues, "Place some zeros next to the decimal point." How many? "A few." He continues, "Eight can't go into 5, so 8 into 50." Let us note here that to replace 5 with 50 is a falsehood. "Eight into 50 is 6," he says while writing 6 next to the decimal point in the quotient. This again is a falsehood. That is six-tenths, not 6. "Eight times 6 is 48." Another falsehood, that's not 48. "Forty-eight from 50 is 2." That's not 2. The work now appears

on the blackboard as follows:

```
   .6
  ____
8)5.000
  4 8
  ---
    2
```

He continues, "Bring down a zero." Why? "That's how you get the right answer." Have you ever heard a meaningful explanation for "bring down"? "Eight into 20 is 2," he says while writing 2 in the hundredth's place of the quotient. Another falsehood, that's not 2. Need more be said? This dialogue by the teacher is composed entirely of falsehoods and meaningless statements.

Clearly it is impossible for a pupil to exercise his/her capacity as a competent contextual learner when taught long division or transformation of a fraction to a decimal (using division) by means of traditional techniques. The author recalls that as a youngster he "broke through" in his attempt to master long division soon after it suddenly occurred to him: "It is not possible to understand that stuff." How strange! It was necessary to "abandon reason" as a prerequisite to mastering the skill. It was impossible to see, contextually, how each "step" in the algorithm was related to the objective of the long division exercise (Barrett, 1990). In fact, we had no choice but to suppress that part of our psyche which, by virtue of how we are created, yearns for meaning and understanding. That part of our

psyche has a very significant name in education. It is called intelligence. We suppressed intelligence in order to "learn" the traditional long division algorithm! (Barrett, 1989) This statement also applies to the learning of all arithmetical algorithms as traditionally taught.

Much research in cognitive psychology supports my contention. Early work by Garner (1966) brought about the conclusion that

> People do perceive properties of sets of stimuli, and these properties affect ease of learning. But the stimuli must be presented so that it is clear to the subject what constitutes a single class or group or subset. If the stimuli are presented so that the subject must learn them as individual stimuli, he can do so, but then he cannot take advantage of some facilitating properties of sets. (p. 16)

Many other researchers (Ausubel, 1968; Battig, 1968; Rohwer and Levin, 1968) have concluded that it is not the repetition of discrete facts that leads to efficient learning but, rather, the coherence of presentation within a meaningful context. Therefore, it should not be a surprise to find that researchers, such as Vaidya and Chansky (1980), have demonstrated that children who understand concepts such as classification and seriation perform best on tests of mathematical ability.

The author's response to this problem is being implemented in many school systems. He has modified various algorithms of arithmetic and the dialogues associated with their performance (Barrett, 1979) so that, with huge gains in efficiency, meaning will always precede the acquisition of the various skills and will always be experienced throughout their performance.

The modification of the preceding algorithm for transforming five eighths to an equivalent decimal follows. After placing five as the dividend and eight as the divisor, pupils are made aware that since eight cannot "go into five once," they must move from the ones' place in the quotient over to the tenths' place. However, it is not possible to fill the tenths' place (in the quotient) unless we change the dividend to tenths. Hence we must "exchange" five wholes (symbolized as 5) for fifty tenths (symbolized as 5.0). Now we can say, "Eight into fifty tenths is six tenths." Having said that, we write ".6" in the quotient (decimal point in the quotient directly above the decimal point in the dividend). Continuing, we say, "Eight times six-tenths are forty-eight tenths." We now write forty-eight tenths, symbolized as 4.8, below 5.0 (decimal points "lined up") and subtract forty-eight tenths from fifty tenths. The difference is two tenths, symbolized as .2, and written below 4.8 (decimal points lined up). "We now have to fill the hundredths' place in the quotient." This cannot be done until we change the new dividend,

two-tenths (.2), into hundredths. So we exchange two tenths (.2) for twenty hundredths (.20). Continuing, we say, "Eight into twenty hundredths is two hundredths," and place the digit 2 in the hundredths' place of the quotient. "Eight times two hundredths are sixteen hundredths," so we write .16 appropriately under .20 and subtract. "Sixteen hundredths from twenty hundredths leaves four hundredths." Four hundredths are written symbolically as .04 under .16. Continuing similarly, we must exchange four hundredths (.04) for forty thousandths (.040) in order to "fill" the thousandths' place in the quotient. Since the new dividend is represented numerically as .040, we say, "Eight into forty thousandths is five thousandths," and write the digit 5 in the thousandths' place of the quotient. Five-eighths are equivalent to .625.

Please note that in direct contrast to the traditional algorithm, every statement made in the modified algorithm is true.

The completed example appears below (Barrett, 1979):

```
  .625
8)5.0
 -4.8
  .20
 -.16
  .040
 -.040
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Is there a view of context here? It is possible for any learner to see that the "maximum tenths" of eight which can be subtracted from fifty tenths (5.0) is six (tenths of eight); the "maximum hundredths" of eight which can be subtracted from twenty hundredths (.20) is two (hundredths of eight); and the "maximum thousandths" of eight which can be subtracted from forty thousandths is five (thousandths of eight).

This contextual vision virtually guarantees delivery of meaning prior to the pupils' acquisition of skill in performing the transformation of a fraction to an equivalent decimal. When pupils are taught arithmetical algorithms by means of falsehoods and meaningless statements the opposite occurs: skill precedes meaning. In fact, for the rest of one's life following the acquisition of skill by traditional means, meaning is a rare occurence among the vast majority of individuals.
Whenever children acquire this "contextual view" of an arithmetical algorithm prior to the acquisition of skill, meaning will also be experienced concurrently every time the skill is being performed.

With tremendous gains in efficiency, any arithmetical algorithm can be performed by youngsters in such a manner that they see each step, even each thought, as related to the overall objective of the exercise. Accordingly, the author has made alterations on arithmetical algorithms in order to accomodate the

contextual view; so that competent contextual learners can master mathematics as inevitably as they mastered their native language or retention of stories (Barrett, 1990).

Will this approach guarantee the learning and retention of the algorithms in the same way that context guarantees the learning and retention of a story? Documentation is available with respect to performance of long division (up to six-place dividends and three-place divisors) by average second graders. This was achieved by means of the contextual approach. When taught this topic traditionally, poor retention is observable when pupils consistently forget what to do next. They are unable to string all the steps together. The consistently successful performance of long division by the second graders demonstrated that they had learned and retained the modified algorithm. Success was also achieved in training fifth and sixth grade teachers to deliver competence in Algebra One to their classes. On four separate occasions (in four different years), fifth and sixth grade classes passed, with flying colors, the same state-wide examination in elementary algebra which was traditionally reserved for the brightest ninth grade students. In fact, during the 1976-1977 school year, a sixth grade class, which had mastered algebra in fifth grade, went on to cover eighty percent of the traditional Intermediate Algebra and Trigonometry curriculum.

Consider what happened every time we were given long division homework. The "inner voice," which we all own, "talked" us through the various steps. This means that all those falsehoods and meaningless statements were repeated by the inner voice over and over again. Consider, also, that this phenomenon continues to be experienced in the performance of many other skills involving operations on fractions and decimals. This is undesirable since the most appropriate functionings of the mind for learning and doing mathematics are very different from those "hammered" into us for the purpose of "learning" arithmetic!

Prior to our encounter with long division, much of arithmetic made sense. That "three plus two equals five" made sense if we could make one group out of two and count all the objects. That "four times five equals twenty" made sense if we could make one group from four groups of five objects each and count them all. Making sense, at this stage, was consistent with and appropriate to our reality as contextual learners. The contextual view brought comfort and confidence to the learning and doing of mathematics to the same degree as the absence of it brought pain and frustration. We were taken "off track" by noncontextual arithmetical algorithms and classroom dialogues.

The reversal, whereby meaning precedes acquisition of skill and is experienced concurrently as the algorithm is being performed, has profound implications for the teaching of mathematics. Empowered elementary school teachers were able to

deliver extraordinary competence in arithmetic and Algebra One to fifth and sixth grade classes in the low socio-economic communities of Roosevelt and Bedford-Stuyvesant, New York. As a consequence of that fact, the claim is made: all of computational arithmetic can be mastered by all fourth graders. The author must emphasize that the basis for his claim is the reality of his experience, as well as statistical analyses and criterion referenced data (Stolzberg, 1977, 1979, 1981). It is the outcome of intense and sustained "hands on" staff-development interventions over the past eighteen years . Further implications are the following:

1. the tremendous reduction in remediation time and efforts (as well as cost);
2. elementary school teachers, competent contextual learners that they are, learn to understand arithmetic while learning how to teach it contextually; and
3. the enormous amount of time made available for noncomputational math topics.

Many years ago, the mathematics education establishment, responding to its failure to deliver acceptable levels of student competence in the learning of its various mathematics curricula, prescribed the "spiral" curriculum as the means of solving the problem. Unfortunately, the spiral approach to learning the

content of five concentrations (for example, Computation, Problem Solving, Geometry, Probability and Statistics) in one school year is similar to learning five stories by reading page one of the first, followed by page one of the second, and so on through all of them, before proceeding similarly with the remaining pages. It may also be compared to studying five paintings by dividing each into ten strips, then looking over strip #1 of the first, followed by strip #1 of the second, and so on through all of them, before proceeding similarly with the remaining strips. No one reads a story or looks at a painting that way. Learners would not be able to achieve contextual vistas in any one of these stories or paintings. A curriculum void of vistas is a "curriculum void" indeed! Curricula will activate and unleash contextual learners to the extent that each offers "clusters" of dynamically related information and is presented to them with the intention of completing a developmental sequence within one before beginning another.

Teaching mathematics contextually not only implies that the teacher posesses the contextual view; it also means that he/she has the ability to consistently articulate contextually within the subject matter. For example, in finding how many twenty-fourths are equivalent to three fourths, a teacher should never say to new learners of this topic, "Four into twenty-four is six and six times three equals eighteen; so three fourths equal eighteen twenty-fourths." There is very little view of context in this statement. Consider the contextuality of the following dialogue:

"Twenty-fourths are which members of the fourths' family?" Students respond, "The sixth." "So the sixth numerator in the family is...?" Students respond, "Eighteen." Internalization of these contextual questions by the learner's inner voice, for repeated applications to such examples in the future, enhances both the retention of and appreciation for the related subject matter. For the purpose of facilitating efforts in cooperative learning, students who articulate subject matter contextually communicate more effectively.

The author is convinced that had traditional practice in the teaching of arithmetic not hammered into our psyches a non-contextual acceptance of arithmetic, vast numbers of students and teachers across the nation would not have similarly approached higher levels of mathematics. Our natural endowment for contextual knowledge would have been more likely activated for the learning of mathematics beyond arithmetic if we had the benefit of contextual teaching.

Not only have students been damaged by their intense and demanding exposure to non-contextual arithmetic but, to the extent that teachers tolerate students' attempts to memorize their way through algebra, trigonometry and calculus, mathematics education in this nation has become severely incapacitated. Consider, for example, the small number of high school students who discover the joy of starting with "given" information and, by means of

definitions, postulates and theorems, "triggering off" dynamically related information until a logical pathway to the solution is found. Consider, again, that among the small percent of students nationwide who survive to the level of trigonometry, only a very small number acquire the satisfaction of seeing the contextual development which derives, for example, the formula for the cosine of a difference of two angles. Consider, finally, that among the very small percent of our student population which survives mathematics to the level of calculus, the majority, in spite of a mechanical competence at determining various limits, do not understand the limit concept and are totally lost with respect to epsilon-delta discussions.

As stated in a previous article of limited circulation which has introduced my methodology to some school systems (Barrett, 1990),

> we must consistently present an internal contextual view of mathematics to learners. This view focuses on the internal dynamics of relationships within the subject matter and is at least as important as the view which sees mathematics in its external contextual relationship to the world. It is the predominant means whereby all learners

everywhere will acquire competence in mathematics, just as it was the means whereby they learned their native language or retained stories.

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